EXAMPLES

Presented below are examples of pipeline lowering situations which illustrate how to use the analytical techniques described in this report. The common procedure used in each case is based upon the previous section of this report and is summarized in Table 6.

Example 1

Example 1 probes one of the simplest possible lowering situations, one in which no special terrain, soil, or pipeline history problems exist. Also, in this example the pipeline operator has elected to avoid lifting to determine the initial axial stress and he has chosen to keep the total longitudinal stress at or below 54 percent of SMYS. The only complicating factor is that the pipeline is postulated to have initial elastic curvature to illustrate the use of transition zones in the lowering profile.

Step 1, Determining Pipeline Parameters

As shown in Table 6, Example 1 involves a 30 inch OD by 0.375-inch wall API Grade 5LX52 natural gas pipeline which operates at 936 psig (72 percent of SMYS). It has experienced no unusual problems in its history and is basically sound. No valves, fittings or field bends exist in the region to be lowered, and the target lowering depth is 10 feet beneath the present location under the center line of a proposed new road.

Step 2, Determining Terrain and Soil Parameters

The terrain near the proposed crossing is flat to gently rolling so no unusual forces are acting on the pipeline due to slope instability or landslides. The soil is found to be cohesive and the permeability is such that the ditch can be pumped dry easily. Because the area never gets so saturated that the soil becomes liquified and because the ditch will not be filled with water, there is no need to apply extra weight to provide negative bouyancy.

TABLE 6. STEPS IN LOWERING AS APPLIED TO SAMPLE PROBLEMS

Step	Description	Example 1	Example 2
1	Determine Pipeline Parameters		
	Required Deflection, H	10 feet	10 feet
	Pipe diameter, wall thickness, grade	30-inch OD by 0.375-inch wall, Gr. X52	16-inch OD by 0.250-inch wall, Gr. X52
	Maximum operating pressure	936 psig	1170 psig
	Operating history (i.e., leaks, girth weld defects, etc.)	No unusual problems	No unusual problems
	Product	Natural gas (specific gravity = 0.1)	Crude Oil (specific gravity = 0.8)
	Bends, Valves, Fittings	None in the area	None in the area
2	Determine Terrain and Soil Parameters		
	Terrain	Gently rolling	Gently rolling
	Soil type	Cohesive, stable	Noncohesive (sand & gravel)
	Ground Water	Trench can be dried by pumping No weights required	Well drained No weights required
	Initial Profile	See Figure 21	See Figure 22
3	Preliminary Calculations		
	Assume initial axial stress,	= 20,000 psi based upon pressure and differential temperature Equations 2 and 3	= 20,000 based upon pressure and and differential temperature Equations 2 and 3
	Check Free deflection Span, L	Table 1, L = 868 ft	Table 1, L = 857 ft
	Check max. stress	Table 1 a = 75.3 ksi	Table 1 a = 75.5 ksi
	Calculate min. contoured span, L	Table C-1 gives L = 1.324 ft.	Table C-1 gives L = 1,179 feet
	Calculate max. stress	Table C-2 gives = 28.1 ksi	Table C-2 gives = 28.1 ksi
	Calculate support spacing, S for maximum stress = 54% SMYS	Table C-3 gives S = 138 ft. differential support height of 3 inches and a temporary axial stress of 15,000 psi due to lowered pressure	Table C-3 gives S = 67 ft. differential support height of 3 inches and a temporary axial stress of 15,000 psi which could be achieved by lowering pressure to near zero.
	Plot lowered profile on initial profile	See Table C-4 and plot on Fig. 21 with H = 10 feet	See Table C-4 and plot on Fig. 22 with H = 10 feet
	Check gross buckling	Not applicable for $\sigma_L \geq 0$	Not applicable for $\sigma_L \geq 0$
	Examine local buckling limit	Not a problem for the contoured trench profile	Not a problem for the contoured trench profile

TABLE 6. (Continued)

Step	Description	Example 1	Example 2
4	Decisions Based on Initial Review		
	Choice of Trench Profile	Use stress-controlled contoured span Span Length with Transition Zones, **1410** ft. Number of Supports Required **= 1410/138 - 1 = 10** Support Spacing, **1410/11** = **128** ft. Support Load Capacity = **128** x (weight per foot) = **19,059** lb.	Use stress-controlled contoured span Span Length with Transition Zones, **1290** ft. Number of supports required **= 1,290/67 - 1 = 19** Support Spacing, **1290/20** = **64.5** ft. Support Load Capacity = **64.5** x (weight per foot) = **6,966** lb.
	To measure existing stress:	No, assume = 20,000 psi	Yes, stress will be measured
	Trench Type	Excavate bell holes for supports or leave soil pillars. Use vertical drop technique.	Excavate bell holes for supports. Use vertical drop technique.
5	Initial Field Considerations		
	Reduce Pressure (Equation **1)**	P = **0.66** Yt/D = **429** psig	**P** = **0.66** Yt/D = **536** psig
	Station personnel at valves	Yes	Yes
	Station observer with radio	Yes	Yes
	Trench safety	Heavy equipment distance Personnel in the trench	Heavy equipment distance Personnel in the trench
	Blasting	Not required	Not required
	Toxic or volatile liquid	Not applicable	Not applicable
	Fracture propagation	Acceptable risk with reduced pressure	Not applicable
	Berms for spills	Not applicable	Build berms to contain possible spill
	Bore holes to locate pipe	Yes, confirms profile and checks soil type and water table	Yes, confirms profile and checks soil type and water table
	Evacuation of nearby building	Not applicable	Not applicable

TABLE 6. (Concluded)

Step	Description	Example 1	Example 2
6	Lifting to Determine Existing Stress	Not applicable	
	Excavate 300 feet		150 feet either side of road
	Survey initial elevation		Get elevations at 5 foot intervals
	Inspect central welds		Remove coating, inspect
	Calculate allowable stress		Allowable is 85% SMYS = 44.2 ksi
	Calculate lift height		Table C-5 suggests 6-inch max.
	Lift and measure load		Measured load, 12.500 lb.
	Survey final elevations		Lift-off span, 142 ft.
	Calculate σ_L currently		Current axial stress 10 ksi, (based on Table C5)
	Add amount due to restored pressure		Adding stress from Eqn. 18, σ_L = 15 ksi
	Check		Maximum stress still OK
	Revise L, if necessary		Table C-1, L = 1,051 feet
	Revise lowered profile		Table C-2 gives a = 25.2 ksi
	Revise support spacing, S		Table C-3 gives S = 79 ft.
			Span Length with Transition Zones, 1,084 ft.
			Number of Supports = 1,084/79 - 1 = 14
			Support Spacing = 1,084/15 = 72.3 ft.
			Support Load Capacity = 72.3 x (weight per foot) = 7.808 lb.
7	Excavation, Inspection, Lowering		
	Excavate for Lowering	Use vertical drop method	Use vertical drop method
	Inspection, General	Visually, for breaks in coating	Visually, for breaks in coating
	Inspection, Girth Welds	Visually, for gross defects	Visually, for gross defects
	Repair or Revise L	Not necessary	Not necessary
	Lift and Lower	Remove supports layer by layer starting at one end	Remove supports layer by layer starting at one end.

The initial profile of the pipeline contains some elastic curvature as shown in Figure 21.

Step 3, Preliminary Calculations

For this **X52** pipeline it will be assumed that the maximum existing axial stress is 20,000 psi on the basis of Equations 2 and 3. The pressure contribution to longitudinal stress is calculated by means of Equation 2, and is found to be 11,232 psi. The stress due to the temperature differential from construction to operation as shown in Equation 3 is taken as 7800 psi on the basis of an assumed temperature differential of 40F.

The free deflection span on the basis of an initial axial stress of 20,000 psi and a maximum deflection of 10 feet is L = 868 ft. as determined from Table 1, and the maximum stress is σ = 75.3 ksi as determined from Table 2. Obviously, this level of stress is undesirably high as is almost always the case for the free deflection span. In this lowering situation the operator elects to use a maximum stress of 54 percent of SMYS or 28.1 ksi; therefore, he must go to the minimum contoured trench parameters of Tables C-1 through C-4. Table C-1 gives the span length, L = 1324 ft, and Table C-2 gives the maximum stress, σ = 28.1 ksi, which is acceptable. From the standpoint of temporary support spacing for an initial axial stress, σ_L, of 20,000 psi, a specific gravity of 0.1, and a maximum allowable stress of 28.1 ksi, Table C-3 presents span lengths of 129 feet for uniform support height and span lengths which will cause excessive stress for both 3-inch and 6-inch differentials between adjacent supports. For a 3-inch differential the span length is 107 feet, and a stress level of 31.6 ksi is reached at the supports. For a 6-inch differential the span length is 132 feet, and a stress level of 36.6 ksi is reached. The operator may elect to limit the lowering increment to 6 inches at alternate supports (equivalent to a 3-inch differential) and just make sure that no support is within five feet of a girth weld. Alternatively, if he chooses to lower the operating pressure prior to excavation according to Equation 1, he will find that σ_L is lowered to about 15,000 psi. In this case Table C-3 indicates that a span length of 138 feet is acceptable for a 3-inch support height differential. Note that if the

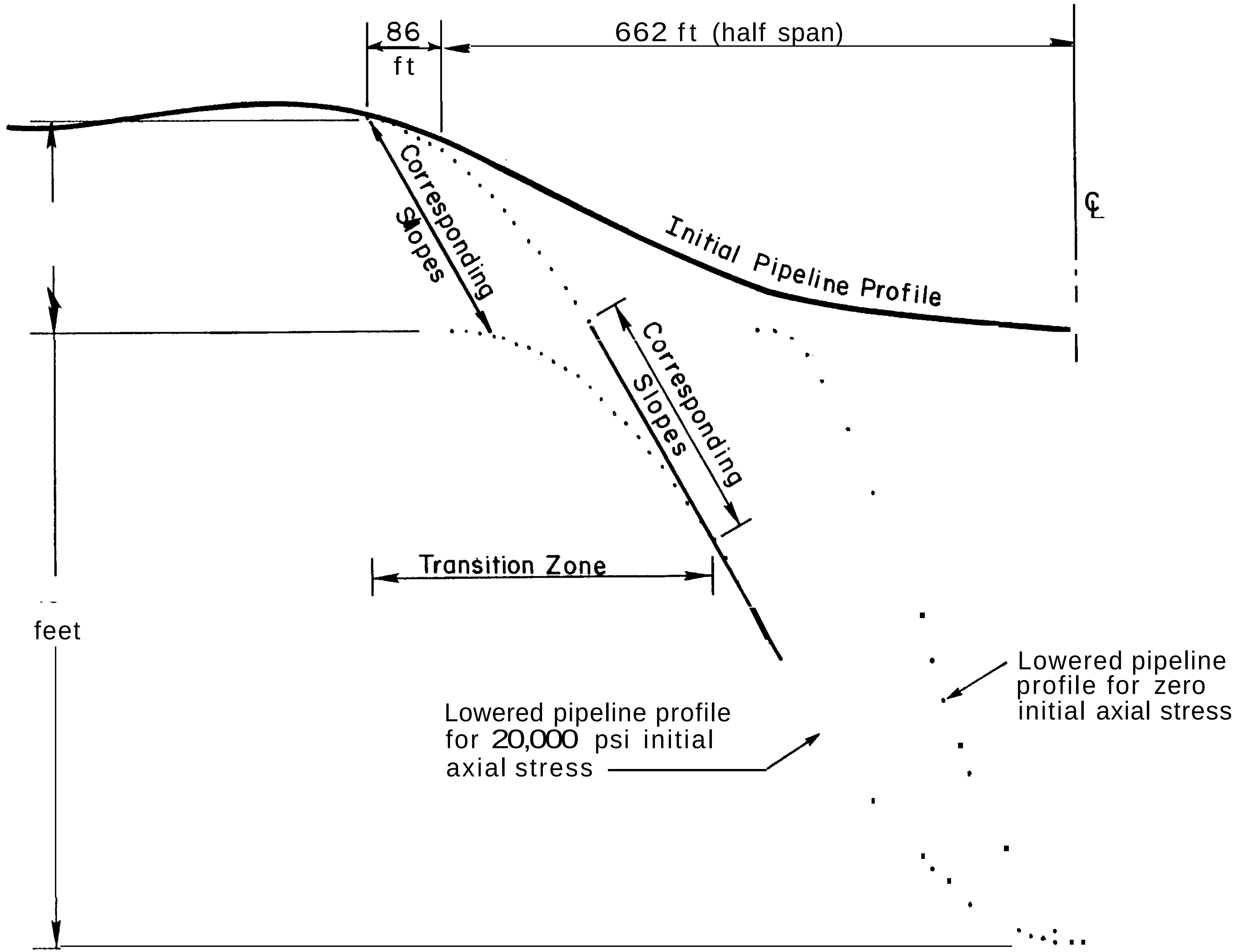

FIGURE 21. PIPELINE PROFILES FOR EXAMPLE 1

operating pressure is not reduced until after the pipeline is sitting on supports, the support spacing for uniform height would have to be 129 feet based upon a 20,000 psi initial axial stress. Finally, the coordinates of the trench profile are obtained from Table C-4 (30-inch pipe, 20,000 psi axial stress, 10-foot maximum deflection) and are plotted in Figure 21. Note that two trench profiles are plotted for illustrative purposes. One, based upon 20,000 psi initial axial stress, is the one needed for the situation of this example; the other, based upon zero axial stress, is presented to show the effect of axial stress. Both are derived from the contoured trench concept.

Because a stress-controlled contoured trench profile is being used, local buckling is not a possibility. General buckling is not possible either because the pipeline is under axial tension.

Because the pipeline is elastically curved a "transition zone" is required as shown in Figure 21 on one end of the section to be lowered to make the pipeline conform to the contoured trench, It is assumed that the other end needs no such transition. The transition profile must be such that the curvature never exceeds that of the contoured profile of Table C-4. This can be handled by plotting the elevations of the bottom of the existing ditch and calculating the existing slope numerically. If the elevations are read from the top of the pipe, it is necessary to subtract the pipe diameter from them to get the elevation of the ditch. The next step involves finding the equivalent slope near the appropriate end of the contoured profile of Table C-4. The contoured profile and existing profile can be smoothly joined at a unique point as shown in Figure 21 (3.5 feet above and 86 feet to the left of the end of the calculated profile). In this case the transition region is formed by prolonging the constant slope portion of the contoured trench until the contoured profile fits the existing ditch. With this done, coordinates of the final ditch to be excavated can be expressed in terms of distance from the centerline versus elevations. As Figure 21 implies of course, the total excavation will exceed the originally calculated value of 1,324 feet due to the transition zone. It is found to be 1,410 feet.

The transition zones for each different lowering situation are likely to be unique. However, each case can be handled in a graphical manner as described above.

Step 4, Decisions Based Upon Initial Review

With the above calculations the pipeline operator can make critical decisions prior to the start of field operations as shown in Table 6. Clearly, it will be necessary to use the stress-controlled contoured trench profile as modified by the transition zone shown in Figure 21. The number of supports or lift points required to temporarily support the pipeline during lowering are calculated by dividing the total span, 1,410 feet, by the minimum spacing between supports, 138 feet, and subtracting 1. Ten equally spaced supports result in eleven equal spans of 128 feet. Thus, ten supports are adequate.

The operator must decide to measure the existing stress or to accept the assumed value. Except in unusual cases where one might expect the stress to be abnormally high in tension (>20,000 psi) or to be compressive, there is no pressing need to measure it. As the next example will show, measuring the stress can be a difficult proposition.

Finally, the operator must decide what kind of ditch to use. This decision is based upon soil type and upon the amount of equipment available. In this example, the soil is cohesive so that the ditch can be vertical sided and remain stable without shoring. The operator could use either the offset trench procedure described in Figure 4a or he could use the vertical drop technique shown in Figure 4b. Since lifting and lowering into an offset trench requires ten sideboom tractors, the operator decides to use the vertical drop method. Note that each support point must support the weight of 128 feet of pipeline* and product (a total of 19,059 lb). The support crib or beam and winch capacity must be designed accordingly.

Step 5, Initial Field Considerations

At the outset of field operations the pressure is reduced to 429 psig on the basis of Equation 1. This not only makes digging around the live line safer, it also lowers the axial stress. The new value of axial stress due to pressure is 5148 psi. Combined with tnat due to the temperature

* The pipeline weight is 119.8 lb/ft and that of the product is 29.1 lb/ft for a total of 148.9 lb/ft.

differential (7800 psi) the total axial stress after the pressure is lowered is 12,948 psi. One advantage of this temporarily reduced stress was noted previously in choosing a temporary support spacing.

The operator must now address other safety issues as well, such as

- Stationing personnel at the upstream and downstream valves to effect closure in the event of an emergency
- Posting an observer with a radio to alert emergency crews and render aid if an emergency arises
- Setting a limit on the distance of heavy equipment from the edge of the trench (based upon soil bearing capacity)
- Stating conditions under which personnel can and cannot enter the trench
- Establishing safety procedures for blasting, if applicable
- Considering wind and weather conditions if a toxic or volatile liquid is involved
- Considering the risk of fracture propagation if the medium is gas (the risk is minimal and acceptable if the pressure is lowered according to Equation 1)
- Creating berms to contain spills in the event of a rupture
- Digging bellholes or boring to locate the pipe accurately
- Evacuating people from nearby buildings if within a zone of danger (from fire, from the blast effects of a rupture, or from a toxic or flammable vapor cloud).

Step 6, Lifting to Determine Existing Stress

This step is not applicable to Example 1 but will be covered in Example 2.

Step 7, Excavation, Inspection, Lowering

In using the vertical drop method ten bell holes on 128-foot centers* will be dug initially to permit the installation of supports. Alternatively, six soil support pillars can be left unexcavated. If support cribs are made of skids or railroad ties, they should be stacked in a manner which will be stable and should be strong enough to support the weight of the pipeline. Also, the pipeline should be prevented from slipping laterally off the supports. Next the pipeline is excavated on both sides and the soil underneath it between the supports is removed. Note that the contoured trench coordinates must be faithfully followed. A surveyor's (spirit) level must be used to assure accurate trench dimensions. Incidentally, the initial support crib bell holes need not be accurately dug as long as they are as deep or deeper than necessary at the given location.

With the pipeline exposed and sitting on skids, a thorough visual inspection should be made. Any pipe or coating damage or corrosion should be repaired as required. Girth welds should be inspected for gross defects. It may be desirable to remove the coating at the welds to conduct a more thorough inspection. As discussed previously, radiography and/or ultrasonics can be used but the welds probably cannot be judged according to the API 1104 Standard. Repairs to welds in a live pipeline are probably limited to installing'full encirclement sleeves. After any necessary repairs have been completed the pipeline is ready for lowering.

Lowering is to be carried out by gradually lowering the support height while not exceeding a 3-inch differential between adjacent supports. For crib-type supports comprised of skids or railroad ties, it is appropriate to start at one end and remove one 3-inch layer at alternate supports (i.e., supports 1, 3, and 5). Next, a six-inch layer can be removed at supports 2, 4, and 6. Then, a six-inch layer can be removed at supports 1, 3, and 5 and so on until the pipeline is lying in the contoured trench. The lifting to remove supports (or to dig out soil pillars, if used) can be done by one or

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* The spacing is based upon eleven spans over ten intermediate supports. Thus, 1410 feet divided by 11 gives 128-foot centers.

more sideboom tractors. The lift distance should not exceed 3 inches in order not to exceed the 3-inch differential limit.

Upon completion of lowering, the pipeline is backfilled and full service is restored.

Before another example is considered, it is worthwhile to refer to Figure 21 again to examine the possible consequences of assuming too high a value of initial axial stress. In Figure 21 the zero axial stress curve is plotted as mentioned previously. Suppose the stress really had been zero. In this case Table C-2 suggests that the maximum stress would reach 22.2 ksi, which is an acceptable value of stress. However, this value arises in conjunction with a much shorter span length (L = 805 feet) as seen in Figure 21. A pipeline of the size of that in Example I having zero initial axial stress, if laid in the 1324-foot trench of Figure 18 would actually have a value of maximum stress much less than 22.2 ksi. This value can be calculated by means of the procedures of Appendix B and is 8.1 ksi. Therefore, it is conservative to assume a higher value of axial stress.

EXAMPLE 2

Example 2 involves a lowering situation in gently rolling terrain where the segment to be lowered contains no bends, valves or fittings. The soil and slopes are stable, and there is no reason to suspect any unusual existing stresses. For the initial review the operator assumes that an axial tensile stress of 20,000 psi is present, but because the line was laid at an ambient temperature near the ground temperature, he suspects that it may be less than 20,000 psi. He intends to measure it by the lift-off method. If it turns out to be lower, he can take advantage of a shorter trench as long as the maximum stress is acceptable. The total maximum longitudinal stress will be limited to 54 percent of SMYS for the final profile, but it will be necessary to allow a higher stress level during the lift-off procedure. This will be handled by setting a limit on the basis of careful inspection of the girth welds in the affected region. It is decided that the tensile stress during lifting can be allowed to reach 85 percent of SMYS if the welds in the affected region are determined to be sound. A precedent for using 85 percent of SMYS exists in the area of offshore pipelines where the 85 percent level is used to limit curvature during the laying operation.

Step 1. Determining Pipeline Parameters

As shown in Table 6, Example 2 involves a 16-inch OD by 0.250-inch wall API Grade 5L X 52 crude oil pipeline which operates at 1170 psig (72 percent of SMYS). It has experienced no unusual problems in its history and is basically sound. The target lowering depth is 10 feet.

Step 2. Determine Terrain and Soil Parameters

The terrain is gently rolling and observations show that the slopes are stable. Therefore, it is suspected that there has been no appreciable movement of the pipeline or the slopes which might cause unusual stresses. The soil is noncohesive and the trench must be excavated with sloping sides in order to avoid shoring. Water is no problem since the area is well drained.

The initial profile of the pipeline (not the soil grade) is shown in Figure 22.

Step 3. Preliminary Calculations

For this X52 pipeline an existing stress of 20,000 psi will be assumed. However, as noted above the operator will attempt to determine the actual level by the lift-off method. A quick check of the free deflection span and maximum stress as listed in Table 6 shows that the resulting stress is unacceptably high as is almost always the case for any appreciable required deflection. Therefore, it is necessary to utilize a contoured trench profile. As shown in Table C-1, the span length for 20,000 psi axial stress for the contoured trench profile is 1179 feet, and as shown in Table C-2 the maximum stress is 28.1 ksi. This level is acceptable since the limit, 54 percent of SMYS, is 28.1 ksi. For an initial axial stress, σ_L, of 20,000 psi, a specific gravity of 0.8, and an allowable stress of 28.1 ksi, the maximum support spacings as shown in Table C-3 are as follows. For zero support height differential, the span length is 65 feet. For a three-inch differential support height the span length is 67 feet but the stress level is 36.7 ksi, greater than the 28.1 ksi allowable value. For the six-inch differential, the span length is 83 feet and the stress level is 44.0 ksi, higher than the 28.1 ksi allowable value. For a realistic differential of 3 inches, then, the operator will have to (1) lower the operating pressure to get the initial axial stress level down some, (2) be careful to place support points away from girth welds, or (3) calculate an acceptable differential height. For the time being since he intends to measure the initial axial stress, he can avoid making any decision, but as shown in Table 6 under Example 2 the spacing is temporarily given as 67 ft. for a 3-inch differential and a temporary axial stress (due to lowering the pressure), of 15,000 psi*. The trench profile coordinates are obtained from Table C-4 (16-inch pipe, 20,000 psi axial stress) and are plotted in Figure 22. Note that again two trench profiles are plotted, one for

* Note that under these conditions the maximum stress in the pipe is 31.8 ksi, greater than the allowable of 28.1 ksi.

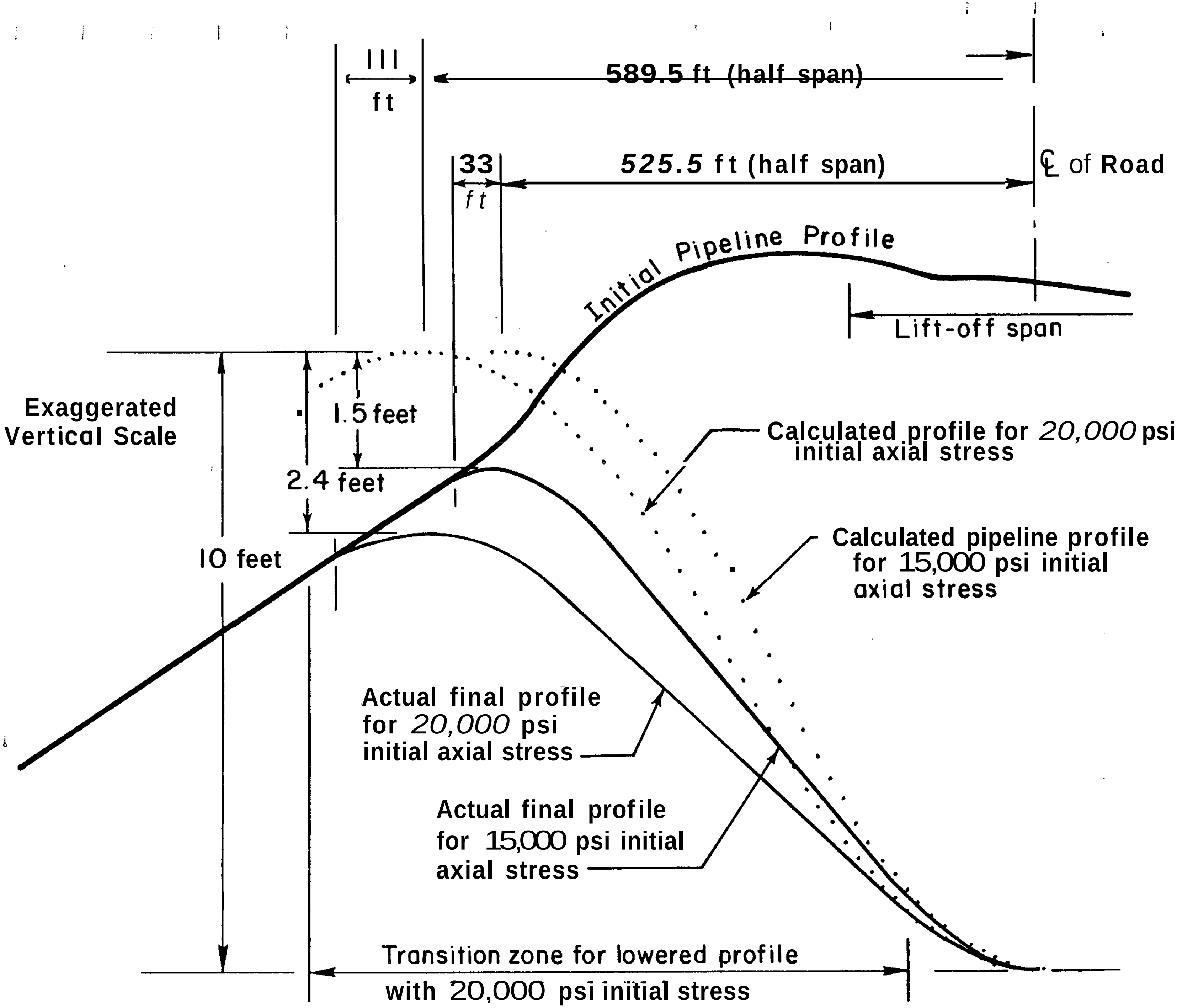

FIGURE 22. PIPELINE PROFILES FOR EXAMPLE 2

20,000 psi axial stress (the initially assumed valve) and one for 15,000 psi (the approximate valve determined subsequently by the lift-off method).

As in Example 1, there is no concern about local buckling because the stress-controlled contoured trench is being used. Also, there is no concern about general buckling since the pipeline is under some amount of tensile load.

It is noted in Figure 22 that a transition zone for one end of the section to be lowered for the assumed 20,000 psi profile is required. This is obtained by plotting the mirror image of the end portion of the curve, finding the point where the slope is parallel to the existing profile, and shifting the contoured profile downward until it is tangent with the existing profile. It is found that this point is 111 feet to the left and 7.4 feet lower than the calculated end point of the profile. By inspection one can then see that the upper and lower portions of the contoured trench may be joined by a straight line segment between points of equal slope. The transition for the 15,000 psi contoured profile is obtained in a similar manner, The end is found to be 30 feet to the left and 1.5 feet lower. In each case the final profile can readily be expressed in terms of elevation and distance coordinates which allow the surveyor to control the excavation.

Step 4. Decisions Based Upon Initial Review

The operator has no real choice but to use a contoured trench profile to keep the stresses within bounds. For the 20,000 psi initial axial stress he determines the number of supports to be 19* by dividing the span length, 1290 feet, by the required support spacing, 67 feet, and subtracting 1. The calculated center-to-center support spacing is 1290/20 = 64.5 feet. As stated previously he intends to measure the existing axial stress by the lift-off method. For this purpose a relatively flat portion of the pipeline 150 feet on either side of the proposed road centerline will be excavated. Finally, the noncohesive soil requires a trench with sloping sides where the soil will control the slope because of its natural angle of repose. Because

* The next largest whole number.

the soil is noncohesive, leaving soil pillars is probably not satisfactory. Instead, bell holes will be excavated for cribs. The vertical drop technique is a necessity because with the offset trench method, the sloping sides of the trench might require more lateral reach than the side boom tractors can accommodate. Note that each support point must support 64.5 feet of pipeline and product (a total weight of **6966** lb). The support crib or beam and winch capacity must be designed accordingly.

Step 5. Initial Field Considerations

At the outset the pressure must be reduced to 536 psig on the basis of Equation 1. The operator must also address the same safety issues covered in Example 1. These are summarized in Table 6. The primary differences between Example 1 and Example 2 are that with crude oil being transported (Example 2) propagating fracture is not a concern and that berms may be necessary to contain an oil spill in the event of an emergency. Also, in Example 2 the cohesionless soil forces heavy equipment to work farther from the pipeline.

Step 6. Lifting to Determine Existing Stress

For the purpose of the lift-off to'determine existing axial stress the operator specifies excavating a 300 foot portion of the pipeline centered on the proposed road crossing and surveying elevations of the top of the pipe at five foot intervals. It is found that three girth welds lie within the interval of 50 feet on either side of the central lift point. The coating is removed from the top quadrant of these welds so that they can be visually inspected. It is not practical to use radiography to inspect these welds in a 16-inch pipeline containing crude oil. The operator may consider ultrasonic inspection as a means of demonstrating that the welds are sound. If the operator is reasonably certain that the welds are sound, the line can be

* The weight of the pipeline is 42.6 lb/ft and that of the product is 65.4 lb/ft for a total of 108 lb/ft.

lifted as long as an upper limit on tensile plus bending stress of 85 percent of SMYS (44.2 ksi) is not exceeded.

Turning to the portion of Table C-5 of Appendix C that deals with this size of pipe and product weight, the operator finds that for an initial axial stress of 20,000 psi the total stress becomes 44.1 ksi for a 6-inch lift. Therefore, the maximum lift height is going to be restricted to about 6 inches. A sideboom tractor equipped with a load cell in line with the lifting hook is used to lift the pipeline 6 inches. As the lift is held, the surveyor quickly rereads the elevations of the top of the pipe at the same five-foot interval locations, and the load is read via the load cell. The surveyor finds that detectable lift-off has occurred at a distance of 71 feet on either side of the lift point for a total lift-off of 142 feet. The load cell indicates 12,500 lb. lifting force. As one can determine in Table C-5 of Appendix C, these valves suggest that the current axial stress at a pressure level of 536 psig is about 10,000 psi. To get the axial stress at the maximum operating pressure, one must add the stress determined by Equation 18 to the 10,000 psi measured value. The result is an initial axial stress of about 15,000 psi at the maximum operating pressure. Table C-1 gives the new span length as 1051 feet, Table C-2 gives the new stress level as 25.2 ksi, Table C-3 gives the support spacing based upon a 3-inch differential and a temporary axial stress value of 10,000 psi as 79 feet, and Table C-4 gives the coordinates for the new profile shown in Figure 22.

Measuring the stress via the lift-off method thus gives a confirmation of the pipeline operator's initial hypothesis that the existing stress was less than 20,000 psi. The operator can now get by with less excavation, fewer support or lift points, and the lower axial stress will permit a 3-inch support height differential.

Step 7. Excavation, Inspection, Lowering

The length of the recalculated span including the one transition zone is 1084 feet as suggested by Figure 22. The number of support points for the lowering operation is $L/S - 1 = 1084/79 - 1 \approx 14$.* This means fifteen spans

* The next largest whole number.

over fourteen intermediate supports on (1084/15) = 72.3 foot centers. Note that each support must be capable of sustaining a load of at least 7808 lb. At this point the operations become identical to those of Step 7 in Example 1 and need not be repeated.

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APPENDIX A

MINIMIZING THE CHANCES OF A LONGITUDINAL RUPTURE IN A PRESSURIZED PIPELINE

APPENDIX A

MINIMIZING THE CHANCES OF A LONGITUDINAL RUPTURE IN A PRESSURIZED PIPELINE

Because pipelines are comprised of relatively thin, relatively ductile steel material, longitudinally oriented defects in them almost invariably initiate failure in a ductile manner. The resulting fracture may accelerate to such speeds that it propagates in a brittle manner or the fracture origin may even appear to be of the cleavage mode. But on a microscopic scale and in terms of failure stress levels, defects in line pipe can be expected to fail a stress levels predictable by a ductile rupture fracture mechanics model. This has been demonstrated in numerous studies supported by the Pipeline Research Committee of the American Gas Association*. As a result it is possible to predict a minimum stress level on the basis of pipe geometry and material properties below which a rupture would not be expected to propagate.

In the paper by Maxey et al., referred to in the footnote, the concept of an arrest stress level is referred to, and an equation is presented for calculating that stress level. In addition, supporting data are presented which make a strong case for its validity. It is shown that when a material exhibits optimum toughness (i.e., its fracture behavior is "flow stress dependent"), the hoop stress level of a longitudinal through-wall defect at failure σ_{hp}, is

$$\sigma_{hp} = \bar{\sigma}/M \qquad \text{(A-1)}$$

where

* Maxey, W. A., Kiefner, J. F., Eiber, R. J. and Ouffy, A. R., "Ductile Fracture Initiation, Propagating and Arrest. In Cylindrical Vessels," Fracture Toughness, Proceedings of the 1971 National Symposium on Fracture Mechanics, Part II. ASTM STP514, American Society for Testing and Materials, 1972, pp. 40-81.

$\bar{\sigma}$ is the flow stress of the material,
(yield strength + 10,000 psi) psi
M is a function of $c/\sqrt{Rt}$
c is half the through-wall defect length
R is the pipe radius, inches
t is the wall thickness, inch.

The empirically determined arrest/propagate boundary indicated by Maxey occurs at approximately $c/\sqrt{Rt} = 3$ which corresponds to an M of 3.33. Using M = 3.33 in Equation A-1, one finds that the hoop stress level below which fracture propagation (i.e., rupture) will not take place is:

$$\sigma_{hp} = \bar{\sigma}/3.33 = .3\bar{\sigma} \qquad \text{(A-2)}$$

If one allows the flow stress, $\bar{\sigma}$, can be conservatively estimated as 1.1Y where Y is the specified minimum yield strength of the material, then the arrest stress level is:

$$\sigma_{hp} = .33Y \qquad \text{(A-3)}$$

In terms of pressure this becomes

$$\frac{PD}{2t} = .33Y$$

or

$$P = .66Yt/D \qquad \text{(A-4)}$$

Equation (A-4) is used as Equation 1 in the text of this report to set a limit on pressure during a lowering operation.

APPENDIX B

DEVELOPMENT OF EQUATIONS FOR DETERMINING EXISTING AXIAL STRESS AND LOWERING INDUCED STRESSES

APPENOIX B

DEVELOPMENT OF EQUATIONS FOR DETERMINING EXISTING AXIAL STRESS AND LOWERING INDUCED STRESSES

THEORETICAL APPROACH

Symbol Definitions

$C_1, C_2, \ldots C_{10}$	Coefficients of deflection equations
E	Elastic modulus of pipe material
f	Fraction of weight used to lower pipeline onto the trench
H	Deflection at center of pipeline
I	Second moment of the cross-sectional area (moment of inertia), with respect to the horizontal centroidal axis
k	Soil stiffness modulus
ℓ	Length of pipeline associated with deflection equations
M_o	End condition moment for contoured trench analysis
N	Axial force acting on the pipeline
P	Lifting force applied at the center of the pipeline
q	Weight per unit length of pipeline and its contents
q_o	Load used to lower pipeline into the trench
T_1, T_2, T_3, T_4	Coefficients of trench profile equations
ω	Pipeline deflection

ω^{i}	$\frac{d\omega}{dx}$
ω^{ii}	$\frac{d^2w}{dx^2}$
w^{iii}	$\frac{d^3\omega}{dx^3}$
ω^{iv}	$\frac{d^4w}{dx^4}$
ω_o	Trench profile
X	Distance from center of pipeline
X_1, X_2, X_3	Local coordinates associated with Regions I, II, and III
α, β, λ	Characteristic lengths
γ	Specific weight of pipe material

General Beam Theory

The pipeline is considered to be an elastic beam of infinite length which is resting on an elastic medium and is subjected to combined axial and transverse loads. Therefore, with this type of model for the pipeline, the deflection and stress equations are based on generalized beam on elastic foundation theory, and the governing differential equation for the elastic response of the pipeline is as follows:

$$EI\omega^{iv} - N\omega^{ii} + k\omega = q$$

This differential equation (or a reduced form, if N and/or k equals zero) remains valid for the pipeline analysis as long as the conditions expressed or implied by the basic assumptions for the beam theory are satisfied.

Theoretical Assumptions

The basic theoretical assumptions associated with the beam on elastic foundation theory are listed below.

1. The beam is initially straight and positioned in a horizontal plane.
2. The deflection angles of the beam are small; thus, $\cos\theta = 1$ and $\sin\theta = \tan\theta = \frac{dw}{dX}$.
3. The deflection due to shear stresses is negligible.
4. The reaction forces on the foundation are vertical at every cross-section (i.e., longitudinal movement of the beam is not restricted by friction).

Furthermore, additional assumptions are made to ensure that the coefficients of the solution equations are constant at every cross-section.

5. The pipe size (diameter and wall thickness) is constant at every cross-section (constant I).
6. The pipe material does not change (constant E).
7. The axial stress is constant over the length of the pipeline (constant N).
8. The pipeline length increase due to deflection of the pipeline adds a uniform axial stress to the pipeline. This stress is constant along the length of the pipeline.
9. The soil is homogeneous and its stiffness is independent of deflection (constant k).
10. The unit weight of pipeline does not vary over the length (constant q).

Loads on the Pipeline

The loads applied to the pipeline (beam) consist of:

1. An axial force, due to (a) the existing axial stress in the pipeline, and (b) the added axial stress due to lengthening of the pipeline upon deflection;
2. The weight of pipe;

3. The weight of the fluid in the pipe;
4. A vertical point load at the center of the pipeline (lifting only); and
5. The vertical reaction forces from the supporting soil.

General Solutions

The form of the general solutions for stress and deflection of the pipeline are the same for (1) lifting the pipeline to determine the existing axial stress and (2) lowering the pipeline by free deflection under its own weight. Only the integration constants are different. The general solution for lowering the pipeline into a contoured trench to reduce the added stress is dependent upon the profile of the trench, hence, the form differs from the free deflection solution.

Free Deflection Solution (Lifting and Lowering)

The free deflection of the pipeline is shown schematically in Figure B-1 for lifting and Figure B-2 for lowering. The form of the solutions for Regions I and II differ, and are dependent upon the sign of the existing axial force in the pipeline. The axial force equals the sum of the axial force and that due to the lengthening of the pipeline upon deflection.

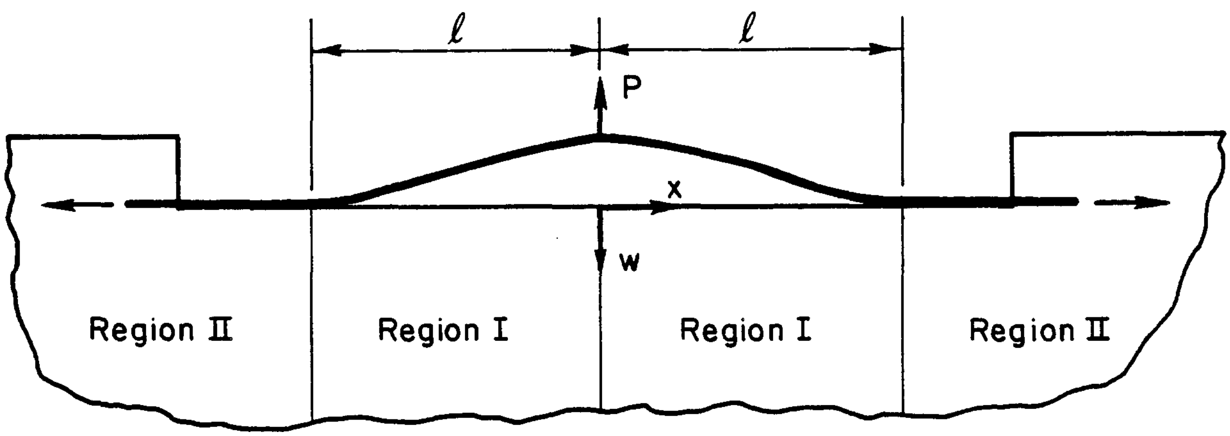

Figure B-1. Pipeline Lifting

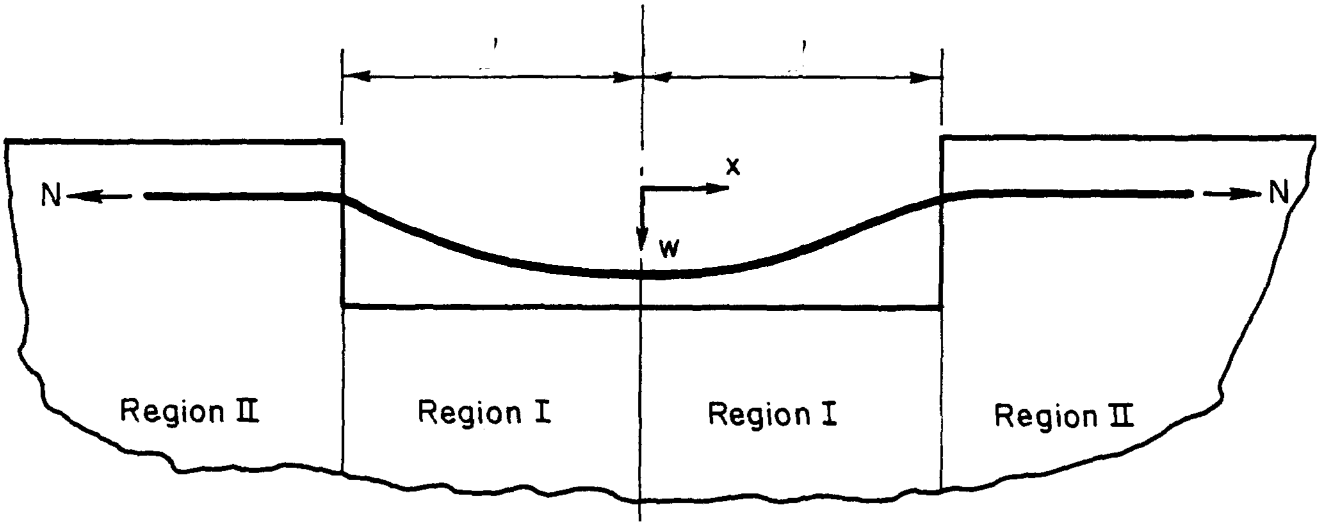

Figure B-2. Pipeline Lowering

Region I

In Region I, the pipeline is not supported by the soil, thus $k = 0$ and the general differential equation reduces to

$$EI\omega^{iv} - N\omega^{ii} = q$$

Using local axes, $x_1 = x$, where $0 \leq x_1 \leq \ell$, with ℓ equal to one-half the total lift-off length for lifting and one-half the length of the unsupported pipe for lowering. For the characteristic length,

$$\lambda = \sqrt{\frac{|N|}{EI}}$$

where $|N|$ is the absolute value of N.

<u>Case 1. N<0</u>

$$\omega_I(X_1) = C_1 + C_2X_1 + C_3\cos(\lambda X_1) + C_4\sin(\lambda X_1) - \frac{q}{2N}X_1^{\;2}$$

$$\omega_I^i(X_1) = C_2 + \lambda[-C_3\sin(\lambda X_1) + C_4\cos(\lambda X_1)] - \frac{q}{N}X_1$$

$$\omega_I^{ii}(X_1) = -\lambda 2[C_3\cos(\lambda X_1) + C_4\sin(\lambda X_1)] - \frac{q}{N}$$

$$\omega_I^{iii}(X_1) = \lambda^3[C_3\sin(\lambda X_1) - C_4\cos(\lambda X_1)]$$

<u>Case 2. N = 0</u> $(EI\omega^{iv} = q)$

$$\omega_I(X_1) = C_1 + C_2X_1 + C_3X_1^2 + C_4X_1^3 + \frac{q}{24EI}X_1^4$$

$$\omega_I^i(X_1) = C_2 + 2C_3X_1 + 3C_4X_1^2 + \frac{q}{6EI}X_1^3$$

$$\omega_I^{ii}(X_1) = 2C_3 + 6C_4X_1 + \frac{q}{2EI}X_1^2$$

$$\omega_I^{iii}(X_1) = 6C_4 + \frac{q}{EI}X_1$$

<u>Case 3. N>0</u>

$$\omega_I(X_1) = C_1 + C_2X_1 + C_3\cosh(\lambda X_1) + C_4\sinh(\lambda X_1) - \frac{q}{2N}X_1^2$$

$$\omega_I^i(X_1) = C_2 + \lambda[C_3\sinh(\lambda X_1) + C_4\cosh(\lambda X_1)] - \frac{q}{N}X_1$$

$$\omega_I^{ii}(X_1) = \lambda^2[C_3\cosh(\lambda X_1) + C_4\sinh(\lambda X_1)] - \frac{q}{N}$$

$$\omega_I^{iii}(X_1) = \lambda^3[C_3\sinh(\lambda X_1) + C_4\cosh(\lambda X_1)]$$

Region II

In Region II, the pipeline is supported by the soil and the general differential equation is applicable.

$$EI\omega^{iv} - N\omega^{ii} + k\omega = q$$

Using local axes, $X_2 = X - \ell$, where $0 \leq X_2 \leq \infty$, the characteristic lengths, α and β are as follows:

$$\alpha = \sqrt{\sqrt{\frac{k}{4EI}} - \frac{N}{4EI}} \qquad \& \qquad \beta = \sqrt{\sqrt{\frac{k}{4EI}} + \frac{N}{4EI}}$$

The form of the solution does not depend on N, and by applying the general boundary condition that the values for w, ω^{i}, ω^{ii}, ω^{iii}, ω^{iv} must be finite as X_2 approaches infinity, we have:

$$\omega_{II}(X_2) = e^{-\beta X_2}[C_5 \cos(\alpha X_2) + C_6 \sin(\alpha X_2)] + \frac{q}{k}$$

$$\omega_{II}^{i}(X_2) = e^{-\beta X_2}[(-\beta C_5 + \alpha C_6)\cos(\alpha X_2) - (\alpha C_5 + \beta C_6)\sin(\alpha X_2)]$$

$$\omega_{II}^{ii}(X_2) = e^{-\beta X_2}[\{C_5(\beta^2 - \alpha^2) - C_6(2\alpha\beta)\}\cos(\alpha X_2)$$

$$+ \{C_5(2\alpha\beta) + C_6(\beta^2 - \alpha^2)\}\sin(\alpha X_2)]$$

$$\omega_{II}^{iii}(X_2) = e^{-\beta X_2}[\{C_5\beta(3\alpha^2 - \beta^2) + C_6\alpha(3\beta^2 - \alpha^2)\}\cos(\alpha X_2)$$

$$+ \{C_5\alpha(\alpha^2 - 3\beta^2) + C_6\beta(3\alpha^2 - \beta^2)\}\sin(\alpha X_2)]$$

Integration Contants

The integration contants are determined by the boundary and matching conditions for the two regions of the pipeline. The boundary and matching conditions for lifting the pipeline at the center to determine the existing axial stress are listed below.

1. $\omega_I(0) = H$

2. $\omega_I^i(0) = 0$

3. $\omega_I^{iii}(0) = \frac{P}{2EI}$

4. $\omega_I(\ell) = 0$

5. $\omega_I(\ell) = \omega_{II}(0)$

6. $\omega_I^i(\ell) = \omega_I^i(0)$

7. $\omega_I^{ii}(\ell) = \omega_{II}^{ii}(0)$

8. $\omega_I^{iii}(\ell) = \omega_{II}^{iii}(0)$

These conditions for lowering the pipeline by free deflection are as follows:

1. $\omega_I(0) = H$

2. $\omega_I^i(0) = 0$

3. $\omega_I^{iii}(0) = 0$

4. $\omega_I(\ell) = \omega_{II}(0)$

5. $\omega_{I}^{i}(\ell) = \omega_{II}^{i}(0)$

6. $\omega_{I}^{ii}(\ell) = \omega_{II}^{ii}(0)$

7. $\omega_{I}^{iii}(\ell) = \omega_{II}^{iii}(0)$

The integration constants were not solved explicitly in terms of the equation parameters, since the large number of complex equations could not easily be reduced to a simple form. However, the equations were reduced to a point where they could be solved numerically using iterative techniques, and the computer coded equations are listed in Appendix D.

Contoured Trench Solution

For the contoured trench analysis, the pipeline is lowered into a trench that has a predefined profile. Reaction forces from the soil act on the pipeline after contact is made with the bottom of the trench. This differs from the previous free deflection analysis, in which the only contact between the pipeline and soil is made beyond the edges of the excavated portion. The differential equation for the pipeline must include this contact, and the following equation is obtained:

$$EI\omega^{iv} - N\omega^{ii} + k(\omega_0) = q,$$

where ω_0 defines the trench profile.

Lowering the pipeline into a trench can be thought of as a two step process. First, the pipeline undergoes a forced deflection so that the profile of the pipeline matches that of the trench. Then, from this position of impending contact, the loads applied to the pipeline to lower it to the trench surface are removed and the pipeline settles into the soil by its own weight. The total response of the pipeline for lowering it into the trench can be determined by superposition of the solutions of these two cases.

Impending Contact Solution

The solution for lowering the pipeline to the trench surface, and, hence, the trench profile, must satisfy the following differential equation.

$$EI\omega_o^{iv} - N\omega_o = q_o$$

Since this equation is the same form as that for Region I of the free deflection analysis, it follows that the solution must take the form of the solutions listed earlier. In this analysis, the pipeline was assumed to be in tension. Tension was assumed, since tension in a pipeline would require a longer length for a given depth of lowering, thus, a pipeline in compression could be conservatively estimated to have a very small tensile stress.

The free deflection lowering of the pipeline results in relatively sharp bends (and high stresses) in the pipeline at the ends of trench, while at the midspan, the larger radius of curvature induces lower stresses. In order to reduce the stresses at the ends, the radius of curvature must be increased in this area. To accomplish this, the trench was divided into two regions as shown in Figure 8-3 (Region III is not excavated and, therefore, is not part of the trench). The profile of the trench was chosen such that Region II is an inverted replica of Region I. The loads shown in the following free body diagram were used to obtain this inverted condition for the deflected pipeline.

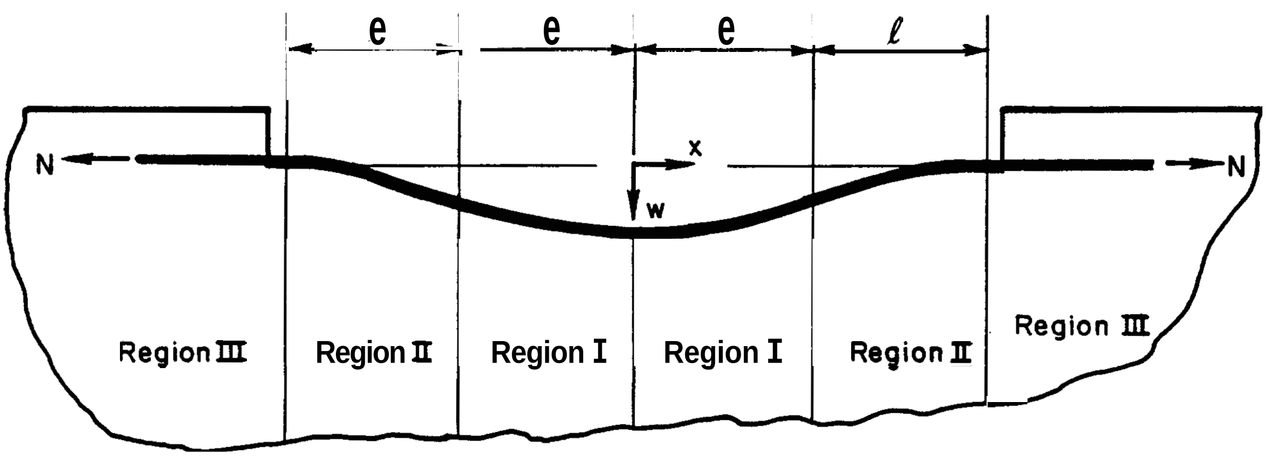

Figure 8-3. Pipeline lowering onto a Contoured Trench.

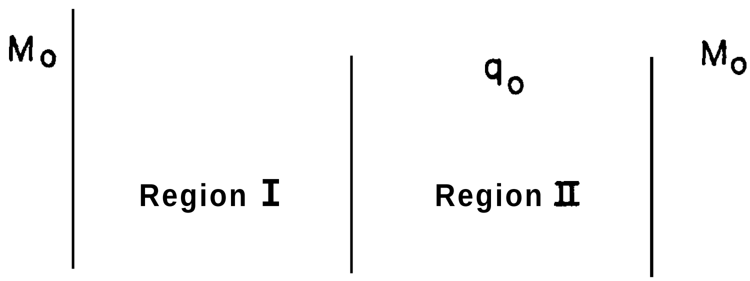

Figure B-4. Free Body Diagram of the Pipeline for Impending Contact with the Trench Surface

The end conditions and matching conditions are as follows:

1. $\omega_{oI}(0) = H$

2. $\omega^{i}_{oI}(0) = 0$

3. $\omega^{iii}_{oI}(0) = 0$

4. $\omega_{oI}(\ell) = \frac{H}{2}$

5. $\omega_{oI}(\ell) = \omega_{oII}(0)$

6. $\omega^{i}_{oI}(\ell) = \omega^{i}_{oII}(0)$

7. $\omega^{ii}_{oI}(\ell) = \omega^{ii}_{oII}(0)$

8. $\omega^{iii}_{oI}(\ell) = \omega^{iii}_{oII}(0)$

9. $\omega_{oII}(\ell) = 0$

10. $\omega^{i}_{oII}(\ell) = 0$

11. $\omega^{ii}_{oII}(\ell) = \frac{M_o}{EI} = \frac{1}{2EI}(q_o \ell^2 - NH)$

12. $\omega^{iii}_{oII}(\ell) = 0$

Using these conditions, and the general solutions for free deflection of the pipeline in tension, the following equations were obtained for the trench contour and the deflection of the pipeline.

Region I

Using local axes, $X_1 = X$, where $0 \leq X_1 \leq \ell$ with ℓ equal to one-quarter the length of the trench.

$$w_{01}(X_1) = T_1 + T_2 \cosh(\lambda X_1) - \frac{q_o}{2N} X_1^2$$

$$w^{i}_{01}(X_1) = \lambda T_2 \sinh(\lambda X_1) - \frac{q_o}{N} X_1$$

$$w^{ii}_{01}(X_1) = \lambda^2 T_2 \cosh(\lambda X_1) - \frac{q_o}{N}$$

$$w^{iii}_{01}(X_1) = \lambda^3 T_2 \sinh(\lambda X_1)$$

Region II

The trench profile for Region II has the same shape as that for Region I, only inverted. Using local axes, $X_2 = X - Q$, where $0 \leq X_2 \leq \ell$.

$$w_{oI}(X_2) = T_3 + T_4 \cosh[\lambda(\ell - X_2)] + \frac{q_o}{2N}(\ell - X_2)^2$$

$$\omega^{i}_{011}(X_2) = -\lambda T_4 \sinh[\lambda(\ell - X_2)] - \frac{q_o}{N}(\ell - X_2)$$

$$\omega^{ii}_{011}(X_2) = \lambda^2 T_4 \cosh[\lambda(\ell - X_2)] + \frac{q}{N}$$

$$\omega^{iii}_{011}(X_2) = -A^3 T_4 \sinh[\lambda(\ell - X_2)]$$

Equation Coefficients

The coefficients of these equations are defined below.

$$T_1 = \frac{q_o}{N}\left[\frac{1}{\lambda^2 \cosh(\lambda \ell)} + \ell^2 - \frac{2}{\lambda^2}\right]$$

$$T_2 = \frac{q_o}{N\lambda^2 \cosh(AQ)}$$

$$T_3 = \frac{q_o}{N\lambda^2 \cosh(\lambda \ell)}$$

$$T_4 = -\frac{q_o}{N\lambda^2 \cosh(\lambda \ell)}$$

and

$$Mo = \frac{q_o}{\lambda^2}\left[1 - \frac{1}{\cosh(\lambda \ell)}\right]$$

$$H = \frac{q_o}{N}\left[\frac{2}{\lambda^2 \cosh(\lambda \ell)} + \ell^2 - \frac{2}{\lambda^2}\right]$$

$$A = \sqrt{\frac{N}{EI}}$$

$$q_o = fq$$

Soil/Pipeline Interaction

The interaction between the soil and the pipeline as the pipeline settles into the soil under its own weight can be determined by applying the following loads to the pipeline:

1. Existing axial stress
2. Added axial stress due to lengthening of the pipeline
3. Weight of the pipeline
4. The opposite of the loads used to obtain the forced deflection
5. Soil reaction forces.

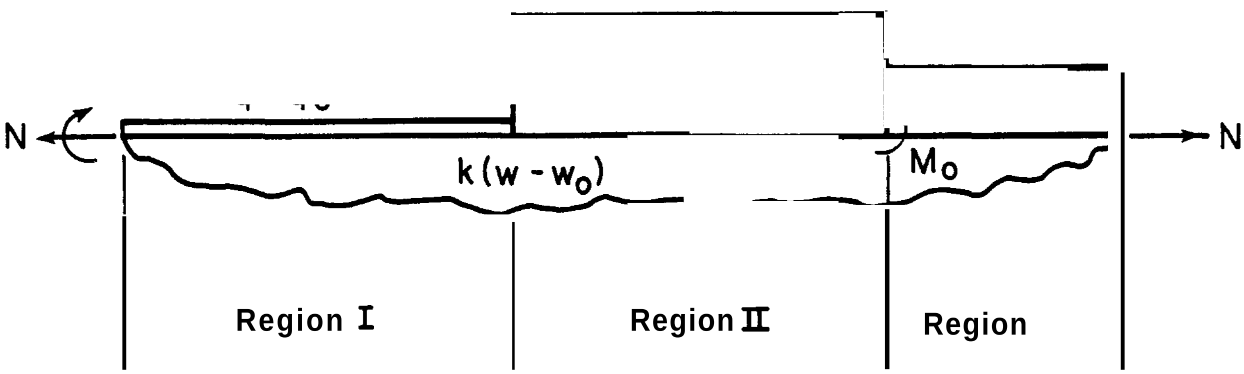

Figure B-5. Free Body Diagram of the Pipeline for Soil/Pipeline Interaction.

For this part of the analysis, the differential equation is

$$EI\omega_1^{iv} - N\omega_1^{ii} + k\omega_1 = q$$

The general solutions of this equation are listed below. The characteristic lengths, a and β, are the same as defined earlier. The trench length parameter, ℓ, is the same as that for the forced deflection.

Region I

$$\omega_{1I}(X_1) = [C_1\cos(\alpha X_1) + C_2\sin(\alpha X_1)]\sinh(\beta X_1)$$

$$+ [C_3\cos(\alpha X_1) + C_4\sin(\alpha X_1)]\cosh(\beta X_1) + \frac{q-q_o}{k}$$

$$\omega_{1I}^{i}(X_1) = [(\alpha C_2+\beta C_3)\cos(\alpha X_1) + (-\alpha C_1+\beta C_4)\sin(\alpha X_1)]\sinh(\beta X_1)$$

$$+ [(\beta C_1+\alpha C_4)\cos(\alpha X_1) + (\beta C_2-\alpha C_3)\sin(\alpha X_1)] \cosh(\beta X_1)$$

$$\omega_{1I}^{ii}(X_1) = [\{(\beta^2-\alpha^2)C_1+(2\alpha\beta)C_4\}\cos(\alpha X_1)+\{(\beta^2-\alpha^2)C_2-(2\alpha\beta)C_3\}\sin(\alpha X_1)]\sinh(\beta X_1)$$

$$+ [\{(2\alpha\beta)C_2+(\beta^2-\alpha^2)C_3\}\cos(\alpha X_1)+\{(-2\alpha\beta)C_1+(\beta^2-\alpha^2)C_4\}\sin(\alpha X_1)]\cosh(\beta X_1)$$

$$\omega_{1I}^{iii}(X_1) = [\{\alpha(3\beta^2-a^2)C_2 + \beta(\beta^2-3\alpha^2)C_3\}\cos(\alpha X_1)$$

$$+ \{\alpha(\alpha^2-3\beta^2)C_1 + \beta(\beta^2-3a^2)C_4\}\sin(\alpha X_1)]\sinh(\beta X_1)$$

$$+ [\{\beta(\beta^2-3\alpha^2)C_1 + \alpha(3\beta^2-a^2)C_4\}\cos(\alpha X_1)$$

$$+ \{\beta(\beta^2-3\alpha^2)C_2 + \alpha(\alpha^2-3\beta^2)C_3\}\sin(\alpha X_1)]\cosh(\beta X_1)$$

Region II

Using local axes, $X_2 = X-\ell$, where $0\le X_2\le\ell$. The form of the equations for ω_{1II} and its derivatives are the same as those for ω_{1I} and its derivatives. These equations can be obtained by making the following substitutions:

$$\omega_{1I} \rightarrow \omega_{1II}$$
$$X_1 \rightarrow X_2$$
$$C_1 \rightarrow C_5$$
$$C_2 \rightarrow C_6$$
$$C_3 \rightarrow C_7$$
$$C_4 \rightarrow C_8$$
$$\frac{q-q_o}{k} \rightarrow \frac{q+q_o}{k}$$

Region III

Using local axes, $X_3 = X-2\ell$, where $0\le X_3\le\infty$.

$$\omega_{1III}(X_3) = e^{-\beta X_3}[C_9\cos(\alpha X_3) + C_{10}\sin(\alpha X_3)] + \frac{q}{k}$$

$$\omega_{1III}^{i}(X_3) = e^{-\beta X_3}[(-\beta C_9+\alpha C_{10})\cos(\alpha X_3) - (\alpha C_9+\beta C_{10})\sin(\alpha X_3)]$$

$$\omega^{ii}_{1III}(X_3) = e^{-\beta X_3}[\{C_9(\beta^2-\alpha^2) - C_{10}(2\alpha\beta)\}\cos(\alpha X_3)$$

$$+ \{C_9(2\alpha\beta) + C_{10}(\beta^2-\alpha^2)\}\sin(\alpha X_3)]$$

$$\omega^{iii}_{1III}(X_3) = e^{-\beta X_3}[\{C_9\beta(3\alpha^2-\beta^2) + C_{10}\alpha(3\beta^2-\alpha^2)\}\cos(\alpha X_3)$$

$$+ \{C_9\alpha(\alpha^2-3\beta^2) + C_{10}\beta(3\alpha^2-\beta^2)\}\sin(\alpha X_3)]$$

Integration Constants

The integration constants are determined by the boundary and matching conditions for the three regions of the pipeline. These conditions are listed below.

1. $\omega^{i}_{1I}(0) = 0$
2. $\omega^{iii}_{1I}(0) = 0$
3. $\omega_{1I}(\ell) = \omega_{1II}(0)$
4. $\omega^{i}_{1I}(\ell) = \omega^{i}_{1II}(0)$
5. $\omega^{ii}_{1I}(\ell) = \omega^{ii}_{1II}(0)$
6. $\omega^{iii}_{1I}(\ell) = \omega^{iii}_{1II}(0)$
7. $\omega_{1II}(\ell) = \omega_{1III}(0)$
8. $\omega^{i}_{1II}(\ell) = \omega^{i}_{1III}(0)$
9. $\omega^{ii}_{1II}(\ell) = \omega^{ii}_{1III}(0) - \frac{Mo}{EI}$
10. $\omega^{iii}_{1II}(\ell) = \omega^{iii}_{1III}(0)$

The integration constants were not solved explicitly in terms of the equation parameters, since the large number of complex equations could not easily be reduced to a simple form. However, the equations were reduced to a point where they could be solved numerically using iterative techniques, and the computer coded equations are listed in Appendix D.

Total Solution

The total solution for lowering the pipeline into a trench can be found by adding the solutions for impending contact and soil/pipeline interaction.

$$\omega = \omega_0 + \omega_1$$

$$\omega^{i} = \omega_0^{i} + \omega_1^{i}$$

$$\omega^{ii} = \omega_0^{ii} + \omega_1^{ii}$$

$$\omega^{iii} = \omega_0^{iii} + \omega_1^{iii}$$

APPENDIX C

TABLES AND FIGURES FOR LOWERING PIPELINES OF VARIOUS SIZES

TABLE C1. SPAN LENGTHS (FEET) FOR MINIMUM CONTOURED TRENCH PROFILE

Maximum Deflection feet	Initial Axial Stress (psi)			
	0	10000	15000	20000
		48 inch pipe		
3				
4				
5				
6				
7				
a				
9				
10				
		30 inch pipe		
3	388 .	491	545	598
4				
5	509	640	709	800 *
6				
7				
a				
9				
10	805	971	1059	1324*

* Longer span than minimum contoured value needed to keep stress within acceptable limits.

TABLE C1. SPAN LENGTHS (FEET) FOR MINIMUM CONTOURED TRENCH PROFILE

Maximum Deflection feet	Initial Axial Stress (psi)			
	0	10000	15000	20000
		30 X 0.375-inch pipe		
3	388	491	545	598
5	509	640	709	800 *
10	805	971	1059	1324 *
		16 X 0.250-inch pipe		
3	341	460	522	580
5	480	621	694	765
10	794	963	1051	1179 *
		8.625 X 0.156-inch pipe		
3	325	452	515	575
5	473	616	690	762
10	793	960	1049	1137

* Longer span than the minimum contoured trench.

TABLE C2. MAXIMUM STRESS (PSI) FOR MINIMUM CONTOURED TRENCH PROFILE

Maximum Deflection feet	Initial Axial Stress (psi) 0	10000	15000	20000
		30 inch pipe		
3	18,547	21,384	24,145	27,508
5	20,717	22,871	25,392	28,100*
10	22,197	25,126	27,699	28,100*
		16 inch pipe		
3	15,414	18,252	21,371	25,121
5	16,125	19,502	22,588	26,230
10	17,824	22,153	25,201	28,100*
		8.625 inch pipe		
3	11,961	16,149	19,722	23,784
5	12,880	17,587	21,052	24,959
10	15,486	20,562	∿24,000**	∿27,500**

* Stress level controlled, span length greater than minimum contoured value.
** Accurate values not possible due to machine limits.

TABLE C-3. MAXIMUM DISTANCE IN FEET BETWEEN SUPPORTS TO LIMIT TOTAL STRESS TO 18,900 psi (54 PERCENT OF SMYS FOR AN GRADE B MATERIAL)*

Initial Axial Stress, L, ksi	Empty Pipeline Sp. gr. = 0 — Differential Support Height, inches: 0	3	6	Gas Pipeline Sp. gr. = 0.1 — Differential Support Height, inches: 0	3	6	Liquid Pipeline Sp. gr. = 0.8 — Differential Support Height, inches: 0	3	6
8-5/8-inch diameter pipe									
0	110	103	94	100	93	83	69	60	56 (24.0)
5	108	94	76 20.6	97	83	73 (21.8)	64	48 (21.3)	60 (28.8)
10	91	65 (20.7)	(20.6)	81	62 (21.5)	85 (26.7)	53	50 (26.3)	64 (33.6)
15	56	70 (25.7)	84 (30.3) (30.3)	51	66 (26.5)	93 (31.6)	34	53 (31.2)	68 (38.5)
16-inch diameter pipe									
0	145	138	128	132	125	113	90	77	73 (24.3)
5	136	121	97 (20.2)	123	106	92 (21.6)	81	62 (21.8)	75 (29.2)
10	112	83 (20.6)	102 (25.2)	101	79 (21.5)	97 (26.5)	65	64 (26.8)	78 (34.1)
15	71	86 (25.5)	108 (30.1)	64	82 (26.5)	102 (31.5)	42	67 (26.8)	81 (39.1)
30-inch diameter pipe									
0	197	189	177	177	168	152	116	94	96 (25.5)
5	179	161	129 (19.9)	159	139	122 (21.6)	101	82 (22.9)	97 (30.5)
10	146	109 20.4	132 (24.9)	129	103 (21.6)	125 (26.6)	81	82 (27.8)	99 (35.5)
15	94	112	136 (29.9)	84	105 (26.6)	128 (31.6)	53	83 (32.8)	101 (40.5)

* Underlined values represent conditions for which no span length exists which will give a stress level at the supports within the acceptable range. Values in parentheses are the stresses at the supports accompanying the corresponding span length and are the minimum possible values.

TABLE C-3. MAXIMUM DISTANCE IN FEET BETWEEN SUPPORTS TO LIMIT TOTAL STRESS TO 28,100 psi (54 PERCENT OF SMYS FOR AN X52 MATERIAL)*

Initial Axial Stress, L, ksi	Empty Pipeline Sp. gr. = 0 Differential Support Height, inches			Gas Pipeline Sp. gr. = 0.1 Differential Support Height, inches			Liquid Pipeline Sp. gr. = 0.8 Differential Support Height, inches		
	0	3	6	0	3	6	0	3	6
8-5/8-inch diameter pipe									
0	155	148	142	138	132	125	88	82	75
5	161	153	143	143	135	125	88	79	60 128.8).
10	156	143	125	138	125	105	83	66	64 (33.6)
15	136	113	(30.3)	120	95	85 131.61	72	53 (31.2)	68 (38.5)
20	101	74 130.61	98 (35.2)	90	70 (31.5)	93 136.5)	55	55 (36.2)	72 (43.3)
16-inch diameter pipe									
0	186	181	175	167	162	156	111	105	96
5	189	181	172	168	160	151	106	97	75 (29.2)
10	180	167	150	159	146	126	98	79	78 (34.1)
15	156	133	108 (30.1)	138	112	102 131.5)	84	67 (31.8)	81 (39.1)
20	118	89 130.5)	113 (35.1)	105	84 (31.5)	107 (36.4)	65	67 (36.7)	83 (44.0)
30-inch diameter pipe									
0	244	239	233	217	212	206	141	132	119
5	238	230	221	210	202	191	131	118	97 (30.5)
10	221	208	191	194	180	157	119	89	99 (35.5)
15	191	166	136 (29.9)	167	138	128 131.61	102	83 132.81	101 (40.5).
20	147	113 130.4)	142 134.91	129	107 131.6)	132 136.6)	79	84 137.8)	102 145.5)

* Underlined values represent conditions for which no span length exists which will give a stress level at the supports within the acceptable range. Values in parentheses are the stresses at the supports accompanying the corresponding span length and are the minimum possible values.

TABLE C-3. MAXIMUM DISTANCE IN FEET BETWEEN SUPPORTS TO LIMIT TOTAL STRESS TO 32,400 psi (54 PERCENT OF SMYS FOR AN X60 MATERIAL)*

Initial Axial Stress, L, ksi	Empty Pipeline Sp. gr. = 0 — Differential Support Height, inches: 0	3	6	Gas Pipeline Sp. gr. = 0.1 — Differential Support Height, inches: 0	3	6	Liquid Pipeline Sp. gr. = 0.8 — Differential Support Height, inches: 0	3	6
				8-5/8-inch diameter pipe					
0	182	176	169	161	154	148	97	92	86
5	190	182	174	167	160	151	99	92	82
10	188	177	165	165	155	142	96	85	64 (33.6)
15	173	157	134	152	136	108	88	67	68 (38.5)
20	144	114	98 135.21	127	94	93 (36.5),	74	55 (36.2)	72 (43.3)
				16-inch diameter pipe					
0	210	204	199	186	181	176	120	115	108
5	216	209	201	190	183	176	118	110	99
10	211	201	189	185	175	163	111	99	78 (34.1)
15	193	178	155	169	153	126	100	76	81 (39.1)
20	162	132	113 135.1)	142	109	107 (36.4)	84	67 (36.7)	83 (44.0)
				30-inch diameter pipe					
0	266	261	256	236	231	226	151	145	136
5	265	258	251	233	226	218	144	134	118
10	254	244	232	221	210	197	133	118	99 (35.5)
15	230	214	192	199	183	153	119	83 (32.8)	101 (40.5)
20	193	163	142 134.9)	168	131	132 (36.6)	100	84 (37.8)	102 145.5)

* Underlined values represent conditions for which no span length exists which will give a stress level at the supports within the acceptable range. Values-in parentheses are the stresses at the supports accompanying the corresponding span length and are the minimum possible values.

TABLE C4. PROFILE OF TRENCH. PIPE SIZE 8 INCHES, DEPTH 3 FEET. INITIAL AXIAL STRESS 0 KSI

Distance from center of span, ft	Trench Depth feet	Distance from center of span, ft	Trench Depth feet
0.00	3.00	110.00	0.67
10.00	2.98	120.00	0.44
20.00	2.90	130.00	0.26
30.00	2.78	140.00	0.13
40.00	2.61	150.00	0.04
50.00	2.40	160.00	0.00
60.00	2.15	163.00	0.00
70.00	1.86		
80.00	1.54		
90.00	1.23		
100.00	0.93		

TABLE C4, PROFILE OF TRENCH. PIPE SIZE
8 INCHES, DEPTH 5 FEET.
INITIAL AXIAL STRESS 0 KSI

Distance from center of span, ft	Trench Depth feet	Distance from center of span, ft	Trench Depth feet
0.00	5.00	130.00	2.05
10.00	4.98	140.00	1.70
20.00	4.93	150.00	1.37
30.00	4.83	160.00	1.07
40.00	4.71	170.00	0.81
50.00	4.54	180.00	0.59
60.00	4.34	190.00	0.40
70.00	4.09	200.00	0.24
80.00	3.82	210.00	0.13
90.00	3.52	220.00	0.05
100.00	3.18	230.00	0.01
110.00	2.81	236.00	0.00
120.00	2.43		

TABLE C4. PROFILE OF TRENCH. PIPE SIZE 16 INCHES, DEPTH 10 FEET. INITIAL AXIAL STRESS 15 KSI

Distance from center of span, ft	Trench Depth feet	Distance from center of span, ft	Trench Depth feet
0.00	10.00	260.00	5.10
10.00	9.99	270.00	4.74
20.00	9.97	280.00	4.39
30.00	9.93	290.00	4.04
40.00	9.88	300.00	3.70
50.00	9.82	310.00	3.39
60.00	9.74	320.00	3.08
70.00	9.64	330.00	2.79
80.00	9.53	340.00	2.51
90.00	9.41	350.00	2.25
100.00	9.27	360.00	2.00
110.00	9.12	370.00	1.77
120.00	8.95	380.00	1.55
130.00	8.76	390.00	1.34
140.00	8.57	400.00	1.15
150.00	8.36	410.00	0.97
160.00	8.13	420.00	0.81
170.00	7.89	430.00	0.67
180.00	7.63	440.00	0.53
190.00	7.36	450.00	0.41
200.00	7.08	460.00	0.31
210.00	6.78	470.00	0.23
220.00	6.47	480.00	0.15
230.00	6.14	490.00	0.09
240.00	5.80	500.00	0.05
250.00	5.45	510.00	0.02
		520.00	0.00
		525.00	0.00

TABLE C4. PROFILE OF TRENCH. PIPE SIZE 16 INCHES, DEPTH 10 FEET. INITIAL AXIAL STRESS 20 KSI

Distance from center of span, ft	Trench Depth feet	Distance from center of span, ft	Trench Depth feet
0.00	10.00	260.00	6.10
10.00	9.99	270.00	5.79
20.00	9.98	280.00	5.48
30.00	9.95	290.00	5.16
40.00	9.91	300.00	4.84
50.00	9.86	310.00	4.52
60.00	9.79	320.00	4.21
70.00	9.72	330.00	3.90
80.00	9.63	340.00	3.61
90.00	9.53	350.00	3.33
100.00	9.42	360.00	3.06
110.00	9.30	370.00	2.80
120.00	9.16	380.00	2.55
130.00	9.02	390.00	2.31
140.00	8.87	400.00	2.09
150.00	8.70	410.00	1.87
160.00	8.52	420.00	1.67
170.00	8.33	430.00	1.48
180.00	8.13	440.00	1.30
190.00	7.91	450.00	1.13
200.00	7.69	460.00	0.98
210.00	7.45	470.00	0.83
220.00	7.20	480.00	0.70
230.00	6.94	490.00	0.58
240.00	6.67	500.00	0.47
250.00	6.39	510.00	0.37
		520.00	0.28
		530.00	0.21
		540.00	0.14
		550.00	0.09
		560.00	0.05
		570.00	0.02
		580.00	0.01
		590.00	0.00

TABLE C4. **PROFILE** *OF* TRENCH. PIPE **SIZE** 30 INCHES, DEPTH 10 **FEET.** INITIAL AXIAL STRESS 0 KSI

Distance from center of span, ft	Trench Depth feet	Distance from center of span, ft	Trench Depth feet
0.00	10.00	270.00	2.29
10.00	9.99	280.00	1.96
20.00	9.95	290.00	1.66
30.00	9.88	300.00	1.38
40.00	9.79	310.00	1.13
50.00	9.67	320.00	0.90
60.00	9.52	330.00	0.70
70.00	9.35	340.00	0.52
80.00	9.15	350.00	0.36
90.00	8.93	360.00	0.24
100.00	8.68	370.00	0.14
110.00	8.41	380.00	0.07
120.00	8.11	390.00	0.02
130.00	7.79	400.00	0.00
140.00	7.45	402.00	0.00
150.00	7.09		
160.00	6.71		
170.00	6.31		
180.00	5.90		
190.00	5.48		
200.00	5.05		
210.00	4.62		
220.00	4.20		
230.00	3.79		
240.00	3.38		
250.00	3.00		
260.00	2.63		

TABLE C4. PROFILE OF TRENCH. PIPE SIZE 30 INCHES, DEPTH 10 FEET. INITIAL AXIAL STRESS 20 KSI

Distance from center of span, ft	Trench Depth feet
0.00	10.00
10.00	10.00
20.00	9.98
30.00	9.96
40.00	9.93
50.00	9.88
60.00	9.83
70.00	9.77
80.00	9.70
90.00	9.62
100.00	9.54
110.00	9.44
120.00	9.33
130.00	9.22
140.00	9.09
150.00	8.96
160.00	8.81
170.00	8.66
180.00	8.50
190.00	8.33
200.00	8.15
210.00	7.96
220.00	7.76
230.00	7.55
240.00	7.33
250.00	7.11
260.00	4.80
270.00	4.40
270.00	4.40
280.00	4.10
290.00	3.70
300.00	3.40
310.00	3.10
320.00	2.80
330.00	2.50
340.00	2.20
350.00	2.00
360.00	1.70

TABLE C4. (Continued)

Distance from center of span, ft	Trench Depth feet
370.00	1.50
380.00	1.30
400.00	0.90
410.00	0.70
420.00	0.60
430.00	0.50
440.00	0.40
450.00	0.30
460.00	0.20
470.00	0.10
480.00	0.10
490.00	0.00
500.00	0.00
510.00	0.00
520.00	0.94
530.00	0.81
540.00	0.69
550.00	0.58
560.00	0.48
570.00	0.39
580.00	0.31
590.00	0.24
600.00	0.18
610.00	0.13
620.00	0.08
630.00	0.05
640.00	0.02
650.00	0.01
660.00	0.00
670.00	0.00

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (8-5/8-INCH OD BY 0.156-INCH PIPE WITH SPECIFIC GRAVITY OF FLUID = 0.8)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
	For a 6-inch lift		
91	1,627	29,958	-10,000
97	2,103	25,234	- 5,000
103	2,548	24,125	0
110	2,962	28,921	5,000
118	3,344	33,719	10,000
126	3,700	38,550	15,000
133	4,033	43,409	20,000
141	4,346	48,291	25,000
148	4,641	53,203	30,000
	For a 12-inch lift		
113	2,206	35,388	-10,000
122	2,882	31,855	-5,000
132	3,503	36,082	0
144	4,075	40,387	5,000
155	4,611	44,826	10,000
167	5,111	49,376	15,000
179	5,580	54,009	20,000
191	6,021	58,714	25,000
202	6,440	63,492	30,000
	For an 18-inch lift		
134	3,101	38,784	-10,000
146	3,817	42,023	- 5,000
159	4,481	45,680	0
173	5,122	49,598	5,000
188	5,733	53,725	10,000
203	6,308	57,986	15,000
217	6,859	62,429	20,000
232	7,383	66,978	25,000
246	7,882	71,607	30,000

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (8-5/8-INCH OD BY 0.156-INCH PIPE WITH SPECIFIC GRAVITY OF FLUID = 0.8) (Continued)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
For a 24-inch lift			
157	4,081	47,455	-10,000
171	4,785	50,569	- 5,000
186	5,460	53,876	0
202	6,126	57,482	5,000
218	6,775	61,349	10,000
235	7,397	65,392	15,000
252	8,004	69,666	20,000
268	8,586	74,072	25,000
283	9,145	78,582	30,000

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (16-INCH OD BY 0.250-INCH PIPE WITH SPECIFIC GRAVITY OF FLUID = 0.8)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
	For a 6-inch lift		
124	8,476	32,020	-10,000
128	9,550	27,098	- 5,000
133	10,578	24,343	0
137	11,568	29,277	5,000
142	12,517	34,216	10,000
148	13,426	39,148	15,000
153	14,299	44,086	20,000
163	15,945	53,988	30,000
	For a 12-inch lift		
149	9,952	39,443	-10,000
156	11,679	34,722	- 5,000
163	13,306	35,798	0
171	14,854	40,541	5,000
180	16,323	45,313	10,000
189	17,719	50,111	15,000
197	19,040	54,910	20,000
206	20,304	59,753	25,000
215	21,507	64,597	30,000
	For an 18-inch lift		
169	11,740	44,203	-10,000
178	13,883	40,835	- 5,000
188	15,891	45,274	0
199	17,782	49,739	5,000
210	19,596	54,315	10,000
221	21,303	58,909	15,000
233	22,928	63,561	20,000
244	24,485	68,283	25,000
256	25,962	73,019	30,000

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (16-INCH OD BY 0.250-INCH PIPE WITH SPECIFIC GRAVITY OF FLUID = 0.8) (Continued)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
For a 24-inch lift			
188	13,969	47,690	-10,000
199	16,290	49,413	- 5,000
211	18,520	53,465	0
224	20,651	57,825	5,000
237	22,661	62,137	10,000
251	24,583	66,548	15,000
264	26,433	71,078	20,000
278	28,1898	75,642	25,000
291	29,870	80,267	30,000

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (30-INCH OD BY **0.375-INCH** PIPE WITH SPECIFIC GRAVITY OF FLUID = -1)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
	For a 6-inch lift		
203	20,017	25,805	-10,000
209	21,875	20,831	- 5,000
215	23,677	16,681	0
221	25,418	21,659	5,000
227	27,103	26,639	10,000
233	28,735	31,620	15,000
239	30,315	36,602	20,000
246	31,846	41,585	25,000
253	33,328	46,568	30,000
	For a 12-inch lift		
242	22,562	31,696	-10,000
251	25,674	26,791	- 5,000
261	28,654	24,123	0
271	31,507	29,044	5,000
281	34,237	33,972	10,000
292	36,848	38,906	15,000
303	39,347	43,840	20,000
314	41,745	48,786	25,000
326	44,042	53,733	30,000
	For an 18-inch lift		
269	24,518	35,785	-10,000
281	28,654	30,981	- 5,000
294	32,588	30,183	0
308	36,318	35,011	5,000
322	39,863	39,858	10,000
336	43,234	44,721	15,000
351	46,434	49,591	20,000
366	49,491	54,483	25,000
382	52,406	59,380	30,000

TABLE C-5. INITIAL AXIAL STRESS IN A PIPELINE AS A FUNCTION OF LIFT-OFF DISTANCE, LIFTING FORCE, AND MAXIMUM STRESS (30-INCH OD BY 0.375-INCH PIPE WITH SPECIFIC GRAVITY OF FLUID = .l) (Continued)

Lift-off Distance, feet	Lifting Force, lb.	Maximum Stress, psi	Initial Axial Stress psi
	For a 24-inch lift		
291	36,603	38,892	-10,000
306	31,564	34,218	- 5,000
322	36,249	35,530	0
339	40,672	40,238	5,000
,356	44,877	44,995	10,000
374	48,a48	49,769	15,000
393	52,612	54,565	20,000
411	56,190	59,387	25,000
429	59,606	64,236	30,000